

Problem Set 1

Question 1 (True or False). Assign “true” or “false” to each of the following statements. For each statement, if you think it is correct, give a brief explanation for why. If you think it is incorrect, give a counterexample.

- If player A in a two-person game has a dominant strategy s_A , then there is a pure strategy Nash equilibrium in which player A plays s_A and player B plays a best response to s_A .
- In a Nash equilibrium of a two-player game each player is playing an optimal strategy, so the two player’s strategies are social-welfare maximizing.

Question 2 (Dominant Strategy and Nash Equilibrium). Two identical firms — let’s call them firm 1 and firm 2 — must decide simultaneously and independently whether to enter a new market and what product to produce if they do enter the market. Each firm, if it enters, can develop and produce either product A or product B. If both firms enter and produce product A they each lose ten million dollars. If both firms enter and both produce product B, they each make a profit of five million dollars. If both enter and one produces A while the other produces B, then they each make a profit of ten million dollars. Any firm that does not enter makes a profit of zero. Finally, if one firm does not enter and the other firm produces A it makes a profit of fifteen million dollars, while if the single entering firm produces B it makes a profit of thirty million dollars.

You are the manager of firm 1 and you have to choose a strategy for your firm.

1. Write down the standard form of this game, with two players, firms 1 and 2, and three strategies for each firm: produce A, produce B or do not enter.
2. One of your employees argues that you should enter the market (although he is not sure what product you should produce) because no matter what firm 2 does, entering and producing product B is better than not entering. Evaluate this argument.
3. Another employee agrees with the person in part (b) and argues that as strategy A could result in a loss (if the other firm also produces A) you should enter and produce B. If both firms reason this way, and thus enter and produce product B, will their play of the game form a Nash equilibrium? Explain.
4. Find all the pure strategy Nash equilibria of this game.
5. Another employee of your firm suggests merging the two firms and deciding cooperatively on strategies so as to maximize the sum of profits. Ignoring whether this merger would be allowed by the regulators do you think its a good idea? Explain.

Question 3 (Guess 2/3 of the Average). Consider the following n -player game. Each player guesses a real number in $[0, 100]$. The player whose guess is closest to the $\frac{2}{3}$ of the average of all guesses wins. Notice that multiple winners are possible. (Hint: IDSDS)

1. Find all Nash equilibria of this game.
2. What if each player can only choose a number in the set $\{1, 2, \dots, 100\}$?
3. What if each player can only choose a number in the set $\{0, 1, 2, \dots, 100\}$?

Question 4 (Nash Equilibria for Three-Player Games). Find all of the mixed-strategy Nash equilibria for the three-player game below. Player 1's strategy set is $\{a, b\}$, player 2's strategy set is $\{x, y\}$, and player 3's strategy set is $\{\ell, r\}$. The first payoff in a cell is for player 1, the second payoff is for player 2, and the third payoff is for player 3.

		Player 3 plays ℓ :				Player 3 plays r :	
		Player 2				Player 2	
		x y				x y	
Player 1	a	7, 1, 5	1, 2, 4		a	5, 0, 5	3, 4, 1
	b	2, 2, 1	5, 3, 2			3, 3, 3	0, 5, 4